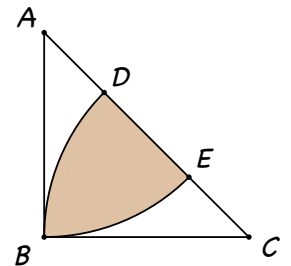# Eighth Annual Upper Peninsula
High School Math Challenge

Northern Michigan University (Marquette, MI, USA)
Saturday 8 April 2017

Team Problems

1. A regular analog clock with continuously moving hour and minute hands starts moving when both hands are together at noon. How many minutes does it take for the minute hand to next catch the hour hand?
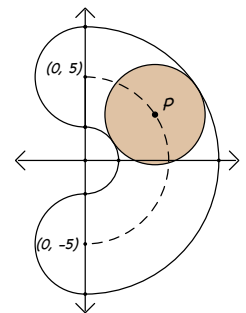
2. Triangle ABC is an isosceles right triangle with BC = AB = 2 inches. Circular arcs of radius 2 inches centered at C and A meet the hypotenuse at D and E, respectively. What is the area of the shaded region?

   (Express your answer in terms of π and/or radicals, if appropriate. Do not approximate as a decimal.)

3. For a given arithmetic sequence, the sum of the first fifty terms is 200, and the sum of the next fifty terms is 2700. What is the first term of the sequence?

4. The center of circle P travels along a semi-circular path from (0, 5) to (0, −5). If the circle sweeps out an area of 39π units², what is the radius of circle P?

5. Two boats traveled in opposite directions (north and south) along the Mississippi River, each at a constant speed. Both boats left at exactly the same time, at sunrise. One boat went from A to B and arrived at 4 pm. The other boat went from B to A and arrived at 9 pm. If they passed each other at noon, what time was sunrise that day?